

Attosecond gamma-ray pulses via nonlinear Compton scattering in the radiation dominated regime

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The feasibility of generation of bright ultrashort gamma-ray pulses is demonstrated in the interaction of a relativistic electron bunch with a counterpropagating tightly-focused superstrong laser beam in the radiation dominated regime. The Compton scattering spectra of gamma-radiation are investigated using a semiclassical description for the electron dynamics in the laser field and a quantum electrodynamical description for the photon emission. We demonstrate the feasibility of ultrashort gamma-ray bursts of hundreds of attoseconds and of dozens of megaelectronvolt photon energies in the near-backwards direction of the initial electron motion. The tightly focused laser field structure and radiation reaction are shown to be responsible for such short gamma-ray bursts which are independent of the durations of the electron bunch and of the laser pulse. The results are measurable with the laser technology available in a near-future.

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Short after the invention of the laser, it was realized that Compton scattering [1] of the laser radiation by a relativistic electron beam can be a bright source of x- and gamma-rays [2, 3]. Later proof-of-principle experiments [4, 5] showed generation of picosecond hard x-rays using electron beams from a linear accelerator. With the appearance of the laser wakefield acceleration technique for electrons [6], an all-optical setup for Thomson/Compton radiation sources from a few hundreds of keV up to 8-9 MeV photon energies, with a shorter duration of about 50 fs, has been demonstrated [7–12]. These experiments are based on linear Thomson/Compton scattering, which produces narrowband gamma-radiation sources for nuclear resonance fluorescence [13]. Recently, a successful effort was accomplished towards Thomson/Compton scattering in the nonlinear regime [14]. In superstrong laser fields Thomson/Compton scattering acquires nonlinear characteristics due to multiple laser photon absorption [15, 16]. Moreover, radiation reaction can enter into play in these extreme conditions [17], which has attracted considerable attention recently [18–26] due to present and next generation petawatt laser systems [27, 28].

In linear Thomson/Compton scattering the duration of the emitted gamma-radiation pulse is determined by the shortest of either the laser or electron beam duration. In an all-optical setup the electron bunch length is of the order of the laser pulse length and the created gamma-rays are of duration of several tens of femtoseconds. Are shorter pulses of gamma-rays necessary? Generally, short laser pulses are required for the time-resolved monitoring and control of fast-evolving processes with the pump-probe technique. The state-of-the-art time-resolution has achieved the attosecond scale by using extreme ultraviolet radiation, which allows to track the dynamics of an

electronic wave packet in an atom [29]. The required frequencies of the short pulses depend on the characteristic energies of the processes under investigation. The molecular dynamics and chemical reactions can be controlled with a few electronvolt excitations driven by an infrared laser field, and the inner-shell electron dynamics by photons with a few 100 eV up to several keV energies. The next challenge is to time-resolve the intra-nuclear dynamics [30, 31]. It is known [32] that typical energies of nuclear single-particle transitions are of the order of 1-10 MeV with typical decay lifetimes of the levels around $10^{-9} - 10^{-15}$ s. The energies of the collective nuclear excitations range from several dozens of keV up to 30 MeV. The disintegration time of compound nuclei during nuclear reactions ranges from $10^{-19} - 10^{-16}$ s. This sets the scale for the required photon energy and pulse duration. There is a wealth of nuclear phenomena for which the investigation of the time resolved dynamics requires short photon pulses, such as, resonance fluorescence (1 fs timescale), resonance internal conversion (1 as timescale) and compound nuclei evolution (zeptosecond timescale).

In this Letter, we investigate the feasibility of generating multi-MeV gamma-rays of several hundreds attoseconds duration via nonlinear Compton scattering of an intense laser pulse by a counterpropagating electron beam. The aim is to produce ultrashort gamma-rays even though using a much longer driving laser pulse and electron bunch. We find an interaction regime when only a small fraction of the electron beam loses sufficient energy due to radiation reaction and is reflected. The reflected electrons emit gamma-rays closer to the laser propagation direction during a short time while leaving the laser focal region, as shown in Fig. 1. The radiation pulse is especially short because the propagation direction of its front part is opposite to that of the rear part of the electron beam (Fig. 1(c)). The scheme relies on the nonlinear regime of interaction, the tightly focused driving laser pulse, and the crucial effect of radiation reaction. All of these three ingredients are necessary to re-

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alise the ultrashort duration of the emitted gamma-rays determined solely by the intrinsic interaction mechanism.

Let us determine the parameters of the applied regime. First of all, we consider the nonlinear regime of Compton scattering when the invariant laser field parameter is large, $\xi \gg 1$, where $\xi \equiv |e|E_0/(m\omega_0)$, E_0 and ω_0 are the laser field and frequency, respectively, and e and m are the electron charge and mass, respectively (Planck units $\hbar = c = 1$ are used throughout). Second, the reflection of the counterpropagating electron in a laser pulse requires a relativistic Lorentz factor $\gamma \approx \xi/2$. In fact, the electron, initially at rest, in the laser field drifts along the laser propagation direction with the Lorentz factor of the drift $\gamma_{drift} = \xi/2$ [34]. Similarly, the electron deviation angle with respect to the laser propagation direction can be estimated as $\theta \sim \xi/\gamma$, and the reflection condition corresponds to $\theta \sim 1$. Third, the interaction has to be in the radiation-dominated regime (RDR), when the radiation losses during a laser period are comparable with the electron's initial energy, and the radiation reaction has a decisive impact on the electron dynamics. This regime is characterized by the parameter $R \equiv \alpha\xi\chi \gtrsim 1$ [17], which is the ratio of the radiated energy during a laser period to the electron energy. Here, α is the fine structure constant, and $\chi \equiv \gamma(\omega_0/m)\xi(1 - \beta \cos \theta)$ the quantum strong field parameter, which determines the recoil of the elec-

tron during the photon emission with $\chi \approx \omega/m\gamma$ [17]. β is the relativistic beta factor of the electron, θ the polar angle between the electron velocity and the laser propagation direction, and ω the emitted photon energy. The RDR is mostly accessible in the quantum regime of interaction when $\chi \gtrsim 1$ [35]. However, the RDR regime is only achievable with extremely intense lasers $\xi \gg 1$. Thus, combining the quantum RDR conditions $R = \alpha\xi\chi \gtrsim 1$ and $\chi \approx 10^{-6}\gamma\xi \sim 1$, with the reflection condition $\gamma \sim \xi/2$, one requires $\gamma \sim \xi \sim 10^3$. Electron beams of GeV energies ($\gamma \sim 10^3$) can be produced by the laser-plasma acceleration technique [6] and the laser intensities of 10^{23} - 10^{24} W/cm² ($\xi \sim 10^3$) are anticipated with next generation facilities (see, e.g., [27, 28]).

Our analysis in this Letter is based on Monte-Carlo simulations employing QED theory for the electron radiation and classical equations of motion for the propagation of electrons between photon emissions [33, 36–38]. In superstrong laser fields $\xi \gg 1$, the coherence length of the photon emission is much smaller than the laser wavelength and the typical size of the electron trajectory [39] (see also [40]). As a result, the photon emission probability is determined by the local electron trajectory. In this case the photon emission probability in the laser field can be approximated by that of constant cross fields with the corresponding local value of the parameter χ (this is the well-known synchrotron approximation) [41], cf. [42]. We employ a linearly polarized focused short laser pulse, which is an approximate solution of Maxwell's equations with first order corrections with respect to the small parameters $(k_0 w_0)^{-1}$ and $(\omega_0 \tau_0)^{-1}$ [26], where k_0 , w_0 and τ_0 are the wave vector, the waist radius and the pulse duration of the laser beam, respectively.

The simulation results for the gamma-radiation properties above the photon energy of 1 MeV are shown in Fig. 2. The applied parameters are the following: the peak intensity of the 4-cycle laser pulse is $I \approx 4.9 \times 10^{23}$ W/cm² ($\xi = 600$), the laser wavelength $\lambda_0 = 1$ μ m, and the laser beam waist size $w_0 = 1$ μ m. The initial kinetic energy of the electrons is 200 MeV ($\gamma_0 = 392$, $\chi_{max} \approx 0.8$). As the electron reflection condition should hold in the laser field, larger initial electron energies $\gamma_0 > \xi/2$ are required because of radiation losses. We employ an electron bunch of length $L_b = 10\lambda_0$, and of transverse size $w_b = w_0$, with the number of electrons $N_e = 3 \times 10^8$. The energy as well as angular spread of the bunch are $\Delta\gamma/\gamma_0 = \Delta\theta = 10^{-3}$.

The time-dependent angular resolved radiation intensity is shown in Fig. 2(a). The radiation sweeps from the polar angle $\theta = 180^\circ$ down to $\theta \approx 20^\circ$. The duration of the emission decreases with decreasing θ . At $\theta \approx 180^\circ$ it is mostly determined by the length of the electron bunch, while at small angles it is smaller than the laser period T_0 . The duration of the gamma-radiation at $\theta = 20^\circ$ with an aperture angle $\Delta\theta = 0.002$ is illustrated in Fig. 2(c). The duration of the main gamma-pulse is about $0.25T_0 = 830$ as. This is the main result of this paper: ultrashort gamma ray bursts of attosecond

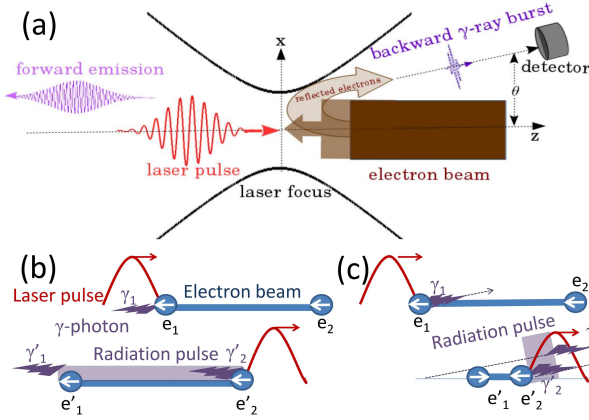


FIG. 1. (Color online) The schematic representation for the generation of ultrashort gamma-ray bursts: (a) The electron beam counterpropagates with the laser pulse and a small slice of the electron beam loses enough energy due to radiation reaction to be reflected and emits ultrashort gamma-rays when leaving the laser focal region. (b) and (c) show how the radiation pulse arises, when the electron beam counterpropagates (b) [or is reflected and copropagates (c)] with the laser pulse. The front part of the gamma radiation “ γ_1 ” is built up when the laser pulse reaches the front of the electron beam “ e_1 ”. The gamma radiation pulse ends “ γ_2 ” when the laser pulse overtakes the end of the electron beam “ e_2 ” (“ γ_1 ” and “ e_1 ” are the positions of the front of the gamma-ray and the electron beam at this moment, respectively). The length of the gamma-ray pulse (γ_1, γ_2) is much shorter in the case of (c), because “ γ_1 ” and “ e_2 ” counterpropagate. See more in the text and in [33].

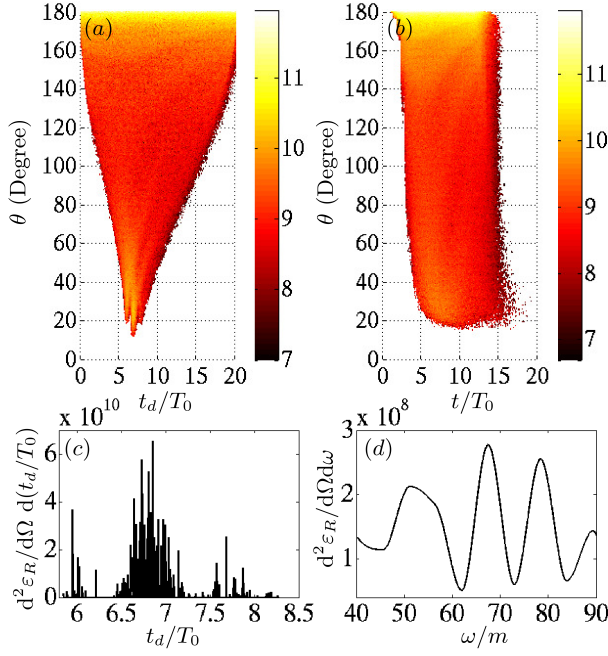


FIG. 2. (Color online) The angle-resolved radiation intensity for photon energies above 1 MeV in a 4-cycle laser pulse with carrier-envelope phase $\phi_{CEP} = 0$ and azimuthal angle of emission with respect to the laser propagation direction $\phi = 180^\circ$ (the spectra are similar at $\phi \approx 0$): (a) Color coded is $\text{Log}_{10}[d^2 \epsilon_R / d\Omega d(t_d/T_0)] \text{ rad}^{-2}$ in the detector time t_d , with the radiation energy ϵ_R in units of the electron rest energy m , the laser period T_0 , and the emission solid angle Ω ; (b) Color coded is $\text{Log}_{10}[d^2 \epsilon_R / d\Omega d(t/T_0)] \text{ rad}^{-2}$ with the electron emission time t ; (c) The differential gamma-ray radiation via $d^2 \epsilon_R / d\Omega d(t_d/T_0)$ at $\theta = 20^\circ$ and $\Delta\theta = 0.002 \text{ rad}$. (d) The spectral distribution $d^2 \epsilon_R / d\Omega d\omega$ of the main pulse in (c). The laser and the electron beam parameters are given in the text.

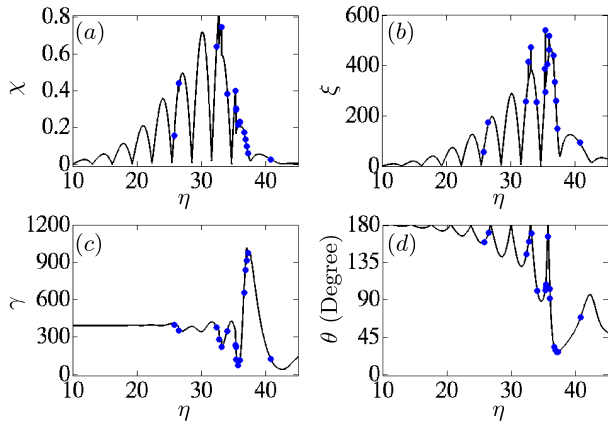


FIG. 3. (Color online) Dynamics of a single exemplary electron with respect to the phase $\eta = \omega_0(t - z)$. Parameters are the same as in Fig. 2. The blue dots present the points where gamma photons are emitted.

duration can be generated closer to the laser propagation direction, while using much longer laser and electron beams (13 fs and 33 fs, respectively, in the given example). The spectral distribution of the gamma ray burst is shown in Fig. 2(d), with the central frequency being $\omega \approx 67m = 34.2 \text{ MeV}$. Narrower gamma-ray pulses can be detected at smaller polar angles where, however, the mean frequency is smaller.

Let us explain the duration of the gamma radiation at different emission angles. The ultrarelativistic electrons in the bunch, which counterpropagate with the laser field, radiate initially in the direction opposite to the laser propagation [see $\eta < 35$ in Fig. 3(d)]. Significant photon emission appears when the χ -parameter achieves a rather large peak value of $\chi \approx 0.6$, as shown in Fig. 3(a). Due to the radiation loss the electron energy decreases (Fig. 3(c)). On the other hand, at this moment the laser field is still large (Fig. 3(b)), yielding the electron reflection at $\eta \approx 36$ [see the large change of θ at $\eta \approx 36$ in Fig. 3(d)]. After the reflection the electron emits briefly closer to the laser propagation direction because it leaves the focal region with an essential decrease of the parameter χ . The emission angle $\theta \sim \xi/\gamma \approx 20^\circ$, is determined by the values of ξ and γ after the reflection. This is the tilting angle of the electron trajectory with respect to the laser propagation direction after the reflection.

The radiation at $\theta \approx 180^\circ$ arises before the electrons reach the reflection point. While the duration of the radiation wave packet of a single electron is extremely short $\Delta t_d^{(1)} \sim \tau_0/4\gamma^2$, each consecutive electron in the bunch contributes into the total radiation field with corresponding time delay. Therefore, the total duration of the emission is of the order of the electron's bunch duration L_b/c . An accurate estimation of the radiation pulse duration yields $\Delta t_d \sim \tau_0/4\gamma^2 + 2L_b/c(1 + \beta)$, and the radiation time of the electron beam $\Delta t = (\tau_0 + L_b/c)/(1 + \beta)$ [33]. Furthermore, $\Delta t_d/\Delta t \approx 1.4$ which corresponds to Fig. 2(a) and (b). The length of the emitted gamma radiation pulse is deduced by calculating the distance between the front of the gamma-pulse (which arises when the laser pulse reaches the front part of the electron beam) and the end of the gamma-pulse (which is determined by the moment when the laser pulse reaches the end of the electron beam) [33] (see also Fig. 1(b) and (c)).

The radiation near $\theta \approx 20^\circ$ arises after the reflection of the electrons. When the laser field is tightly focused, the emission time in this case again is of the order of the duration of the electron beam $\Delta t \approx L_R/c + L_b/c(1 + \beta) \sim L_b/2c$, with the Rayleigh length $L_R = \pi w_0^2/\lambda_0$ [33]. However, due to reflection, the front of the radiation field and the rear part of the electron beam are counterpropagating, which yields a very short radiation pulse: $\Delta t_d \sim \pi T_0 (w_0/\lambda_0)^2 (1 - \beta \cos \theta) + L_b(1 - \cos \theta)/c(1 + \beta) \sim L_b(1 - \cos \theta)/2c$ [33].

Moreover, there is another reason which further decreases the emission time. During the forward radiation all electrons loose energy which facilitates the reflection

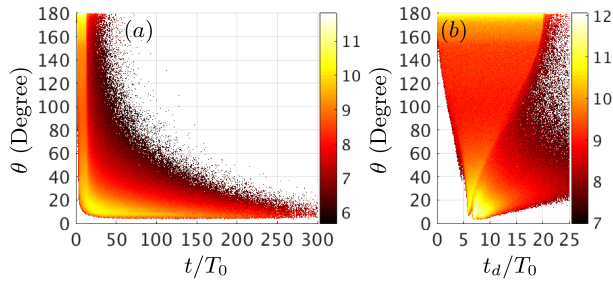


FIG. 4. (Color online) The angle-resolved radiation intensity in a 4-cycle plane-wave laser pulse with photons energies above 1 MeV: (a) $\text{Log}_{10}[\text{d}^2 \epsilon_R / \text{d}\Omega \text{d}(t/T_0)] \text{ rad}^{-2}$ in the electron time; (b) $\text{Log}_{10}[\text{d}^2 \epsilon_R / \text{d}\Omega \text{d}(t_d/T_0)] \text{ rad}^{-2}$ in the detector time. Parameters are the same as in Fig. 2.

when the condition $\gamma \sim \xi/2$ is approached. However, only the front fraction of the bunch can encounter the strongest laser field at the peak of the laser pulse within the focal region and loose enough energy to fulfil the reflection condition. The rear electrons experience a weaker laser field, because of the laser defocusing effect and can not achieve the reflection. Then, the effective length \tilde{L}_b of the reflected electron bunch is shorter than the total bunch length, for the given parameters $\tilde{L}_b \approx 10\lambda_0$ [33]. For the given parameters our estimate provides $\Delta t_d \approx 0.24T_0$ and $\Delta t \approx 4T_0$, which is in agreement with Fig. 2(a) and (b).

The focusing of the laser beam is absolutely essential. In a plane-wave laser pulse, as shown in Fig. 4, the emission time of the radiation before the reflection can be seen from the area $\theta > 90^\circ$ in Fig. 4(a), and that after the reflection from $\theta < 90^\circ$. In the latter case, the electron moves along the propagation direction of the laser pulse and experiences the field of the rest of the laser pulse. Then, the emission of photons takes the long time $\Delta t \sim \tau_0/(1 - \beta) \gg L_b/c$ [33]. In the frame of the detector, the duration of the radiation pulse is shortened

with respect to the emission time by a factor $1 - \beta \cos \theta$ to become $\Delta t_d \sim \tau_0(1 - \beta \cos \theta)/(1 - \beta)$ [33] [see the time range for $\theta < 90^\circ$ in Fig. 4(b) compared to Fig. 4(a)]. However, the shortest duration of the radiation pulse at $\theta \approx 20^\circ$ is still much larger than the laser period, in contrast to the case of the focused field [in the focused field $\tau_0/(1 - \beta)$ is replaced by L_R/c in the emission time].

Several gamma-ray bursts are observable near $\theta \approx 20^\circ$ in the 4-cycle laser pulse (Fig. 2(a)). Moreover, a single gamma-ray burst arises in a 2-cycle laser pulse, and a gamma-ray comb is formed by employing longer laser pulses [33]. This is because the electron can be reflected at any wave crest due to the stochastic character of the gamma-photon emission, while there is only one single burst when stochastic effects are neglected [33]. This feature of the angle-resolved radiation intensity can serve as an indicator of stochastic effects in photon emission.

The described effect of the short gamma ray generation near the reflection condition is rather robust. While here one example of the effect at $\xi = 600$ and $\gamma_0 \approx 400$ has been provided, our simulations show that the same mechanism for ultrashort gamma-ray bursts works as well when varying the laser field and the electron energy within a large range: $\delta\xi/\xi \sim 1/2$ and $\delta\gamma/\gamma_0 \sim 1/2$ [33].

Finally, we estimate the total number of photons in the gamma-ray burst of 830 as duration for the parameters in Fig. 2 to be $N_{ph} \sim 10^2$ within the emission solid angle $\Delta\Omega = 1 \text{ mrad}^2$. In spite of a small number of total photons, the photon flux (F) and the brilliance (B) are rather large due to the short duration of the pulse: $F \sim 10^{14} \text{ photons s}^{-1} 0.1\% \text{BW}$, and $B \sim 3 \times 10^{20} \text{ photons s}^{-1} \text{ mrad}^{-2} \text{ mm}^{-2} 0.1\% \text{BW}$, respectively, e.g., the brilliance is 2 orders of magnitude larger than in the recent experiment [12].

In conclusion, we have shown that brilliant attosecond gamma-ray bursts can be produced by the combined effect of laser focusing and radiation reaction in nonlinear Compton scattering in the radiation dominated regime.

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